Wave Optics

Study Guide

Wave optics Syllabus

Wave front and Huygens principle, reflection and refraction of plane wave at a plane surface using wave fronts proof of laws of reflection and refraction using Huygens principle. Interference, Young's double slit experiment and expression for fringe width, coherent sources and sustained interference of light, diffraction due to a single slit, width of central maximum, resolving power of microscope and astronomical telescope, polarisation, plane polarisation, Brewsters angle, uses of plane polarised light polaroids

Wave optics key points

- Wave front and Huygens principle,
- Reflection and refraction of plane wave at a plane surface using wave fronts
- Proof of laws of reflection and refraction using Huygens principle.
- Interference, young’s double slit experiment and expression for fringe width,
- Coherent sources and sustained interference of light,
- Diffraction due to a single slit ,width of central maximum,
- Resolving power of microscope and astronomical telescope
- Polarisation, plane polarisation,
- Brewsters angle,
- Uses of plane polarised light, polaroids

Light is a transverse electromagnetic wave

Ray optics

- Macro
- Explains reflection
- Refraction
- Formation of images
- Formation of shadows

Wave optics

- Micro
- Explains reflection
- Refraction
- Superposition principle
- Interference
- Diffraction
- Formation of fringes
- Polarisation

18^{th} -19^{th} century work on understanding wave nature of light

- **Huygens Principle**

Wavefront - locus of all the point which oscillate in the same phase , or surfaces of constant phase

Wave length - separation between consecutive wave fronts is a wavelength

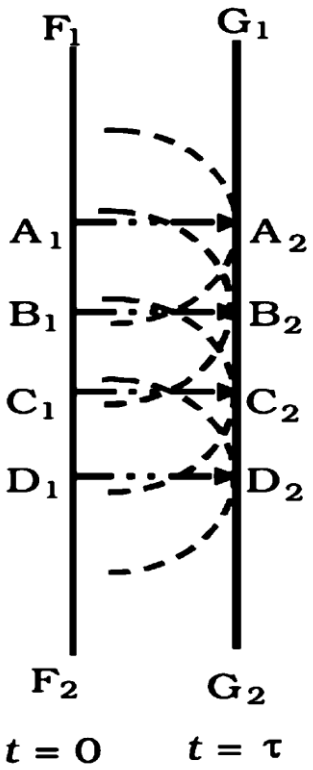
Speed of wave - speed of light or any electromagnetic wave

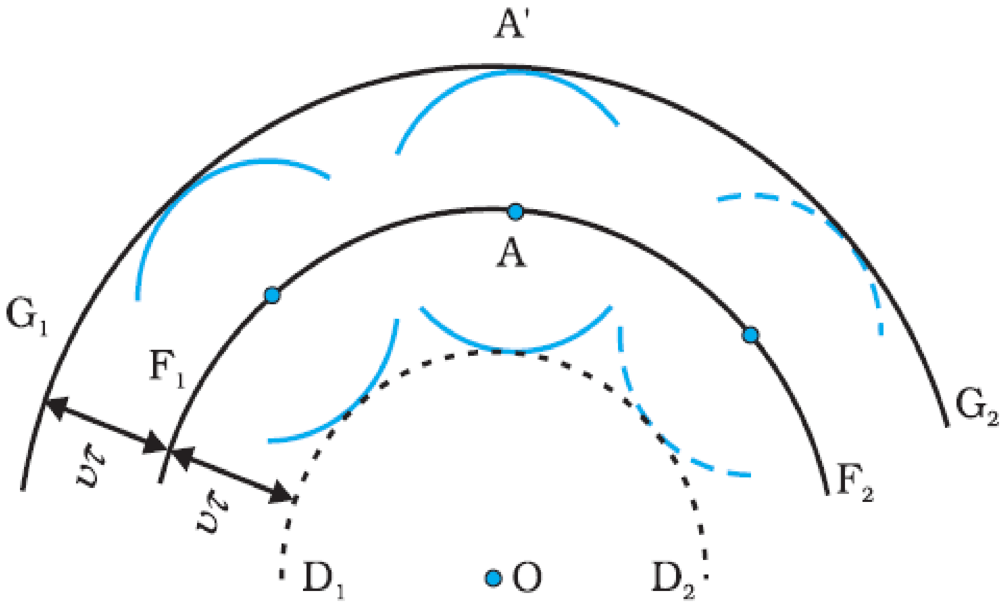
Direction of travel - direction given by rays perpendicular to the wave-front

So..

Huygens principle is essentially a geometrical construction,

- which gives the shape of the wavefront at any time
- allows us to determine the shape of the wavefront at a later time





Watch the animation

Huygens principle –2 rules for constructing a new wavefront

1. Each point on a wavefront is a source of a secondary disturbance and wavelets emanating from these points spread out in all directions with the speed of the wave. The wavelets are called secondary wavelets
2. The new position of the wavefront at an instant is given by the forward envelope of the secondary wavelets (the backward envelop is ignored)

Now draw geometrical wavefronts

Source in homogeneous medium

- Waves travelling in 3 D space
- Spherical wavefront
- Plane wavefront
- Cylindrical wavefront

Imagine the type of source

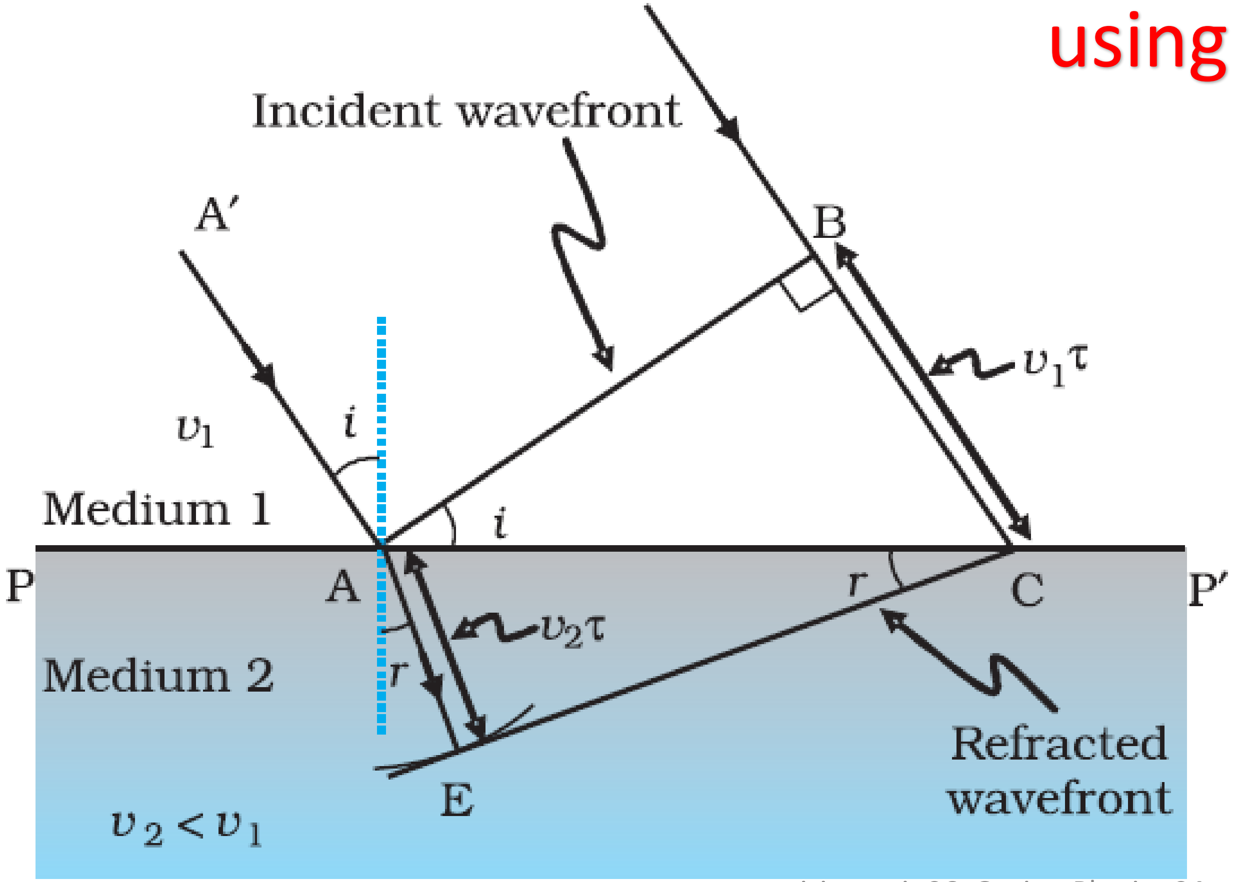
To explain refraction using Huygen's construction

Remember

1. Velocity changes
2. Wavelength changes
3. Frequency remains the same
4. Amplitude decreases depending upon reflection and absorption
5. Laws of refraction are always true

$BC = v_1 \tau$

Study and draw the diagram using a ruler and pencil



$$\sin i = \frac{BC}{AC} = \frac{v_1 \tau}{AC}$$

and

$$\sin r = \frac{AE}{AC} = \frac{v_2 \tau}{AC}$$

Thus we obtain

$$\frac{\sin i}{\sin r} = \frac{v_1}{v_2}$$

$$n_1 = \frac{c}{v_1}$$

and

$$n_2 = \frac{c}{v_2}$$

$$\frac{\lambda_1}{\lambda_2} = \frac{BC}{AE} = \frac{v_1}{v_2}$$

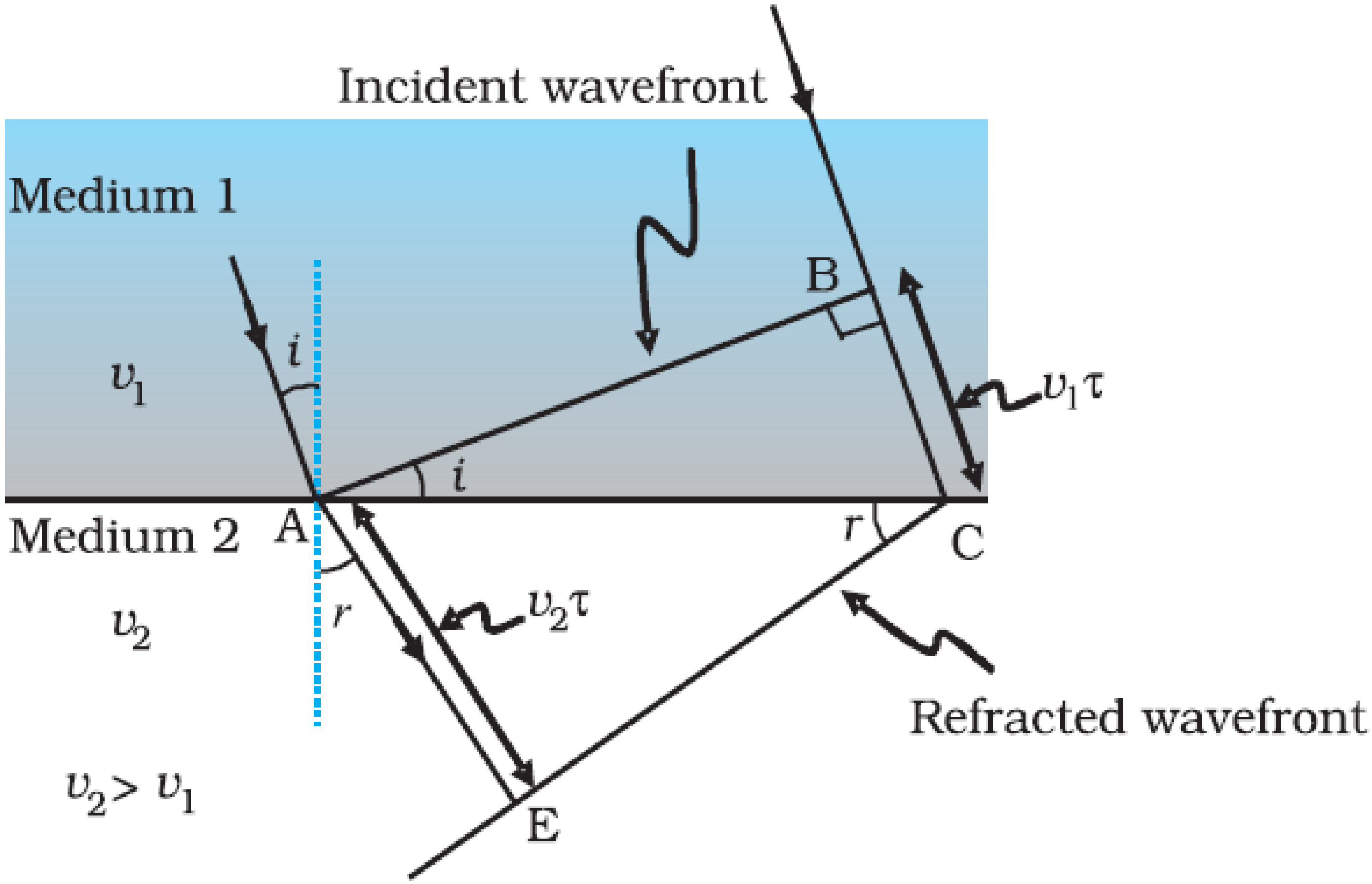
or

$$\frac{v_1}{\lambda_1} = \frac{v_2}{\lambda_2}$$

$$n_1 \sin i = n_2 \sin r$$

Write the expressions

When light travels from denser to rarer medium

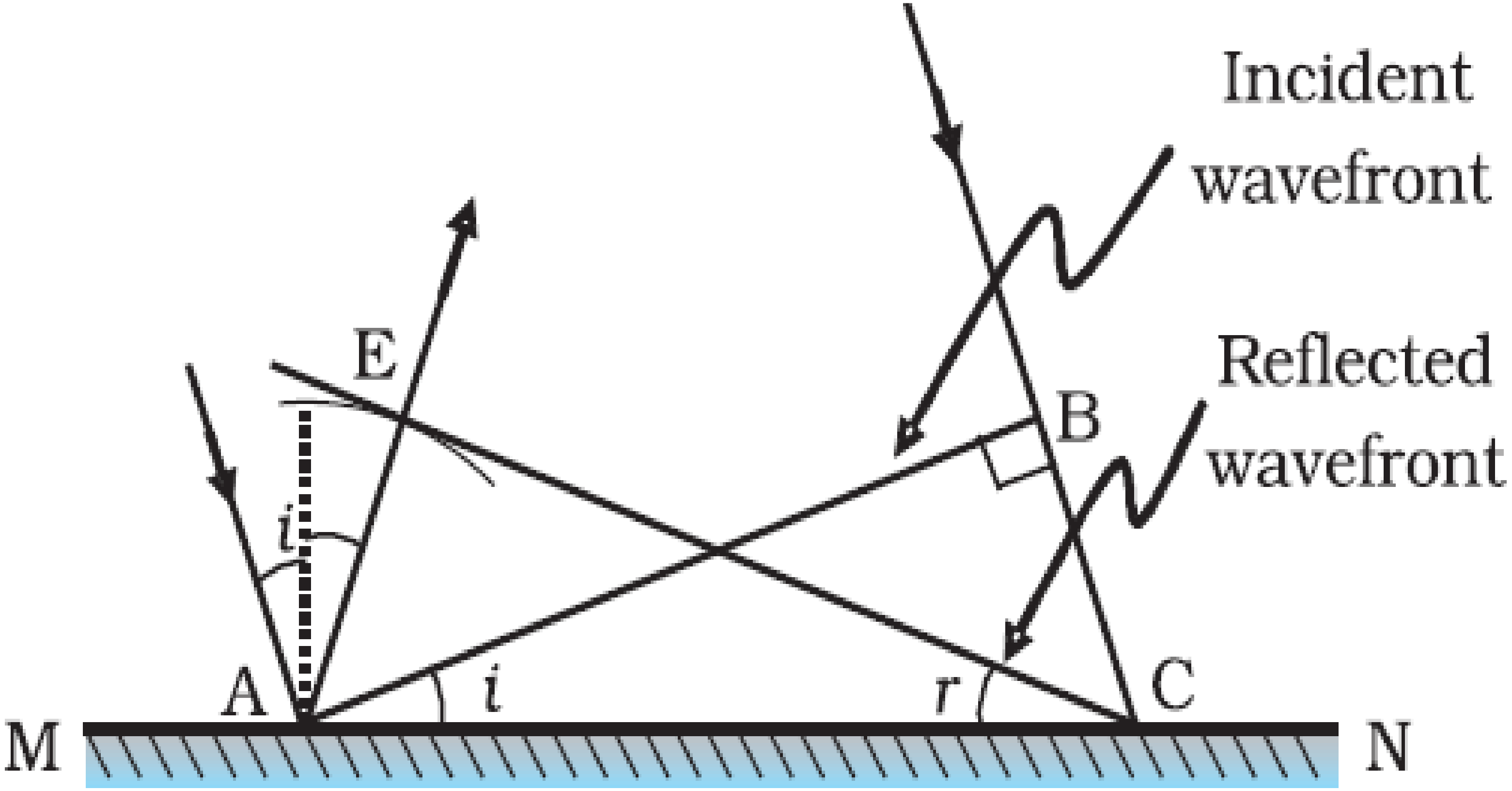


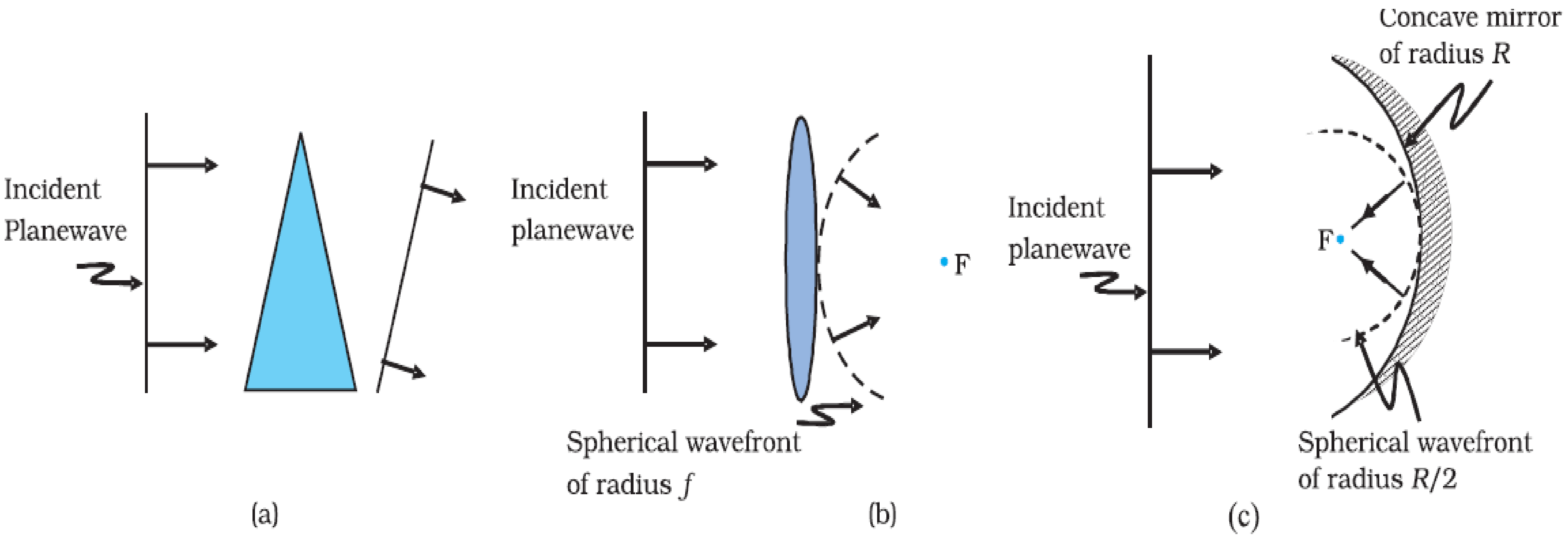
To explain reflection using Huygen's construction

Remember reflection is bouncing back of light in the same medium

1. Velocity remains the same
2. Wavelength remains the same
3. Frequency remains the same
4. Amplitude decreases depending upon refraction and absorption
5. Laws of reflection are always true

Draw this several times





Draw and check these out

Principle of Superposition

1. **The net displacement of any element of the medium at any instant is the algebraic sum of the displacements due to each wave.**

- $\mathbf{y_1 = Asin(\omega_1 t)}$
- $\mathbf{y_2 = Asin(\omega_2 t)}$
- $\mathbf{Y = Asin(\omega_1 t) + Asin(\omega_2 t)}$

The resultant differs from the original

2. **After the region of overlap the two waves continue on their original paths as before**

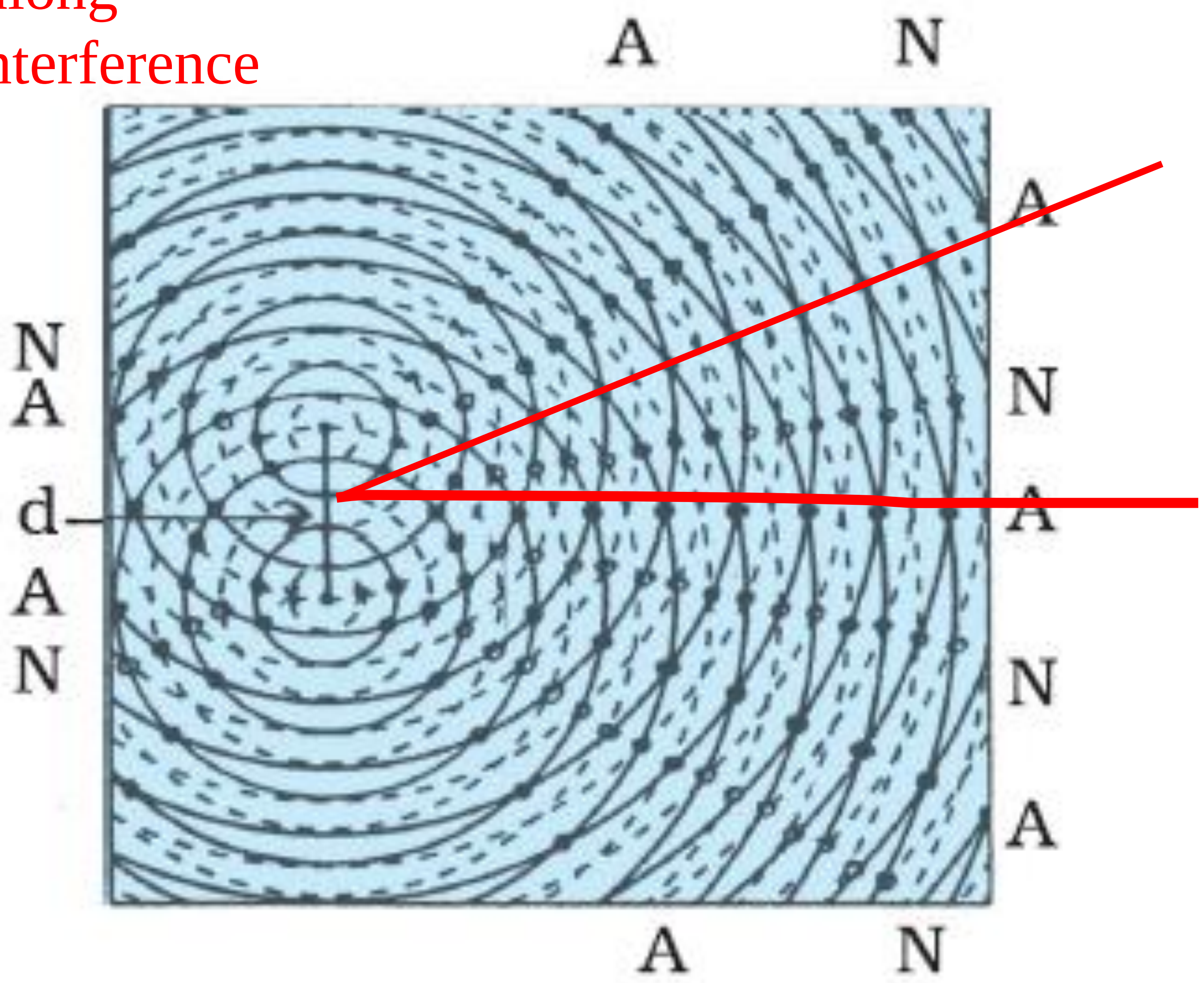
Interference

Conditions

- Two waves of same wavelength
- Same frequency and nature
- Nearly same amplitude
- Same phase

- Redistribution of energy in space
- Light and dark fringes are formed

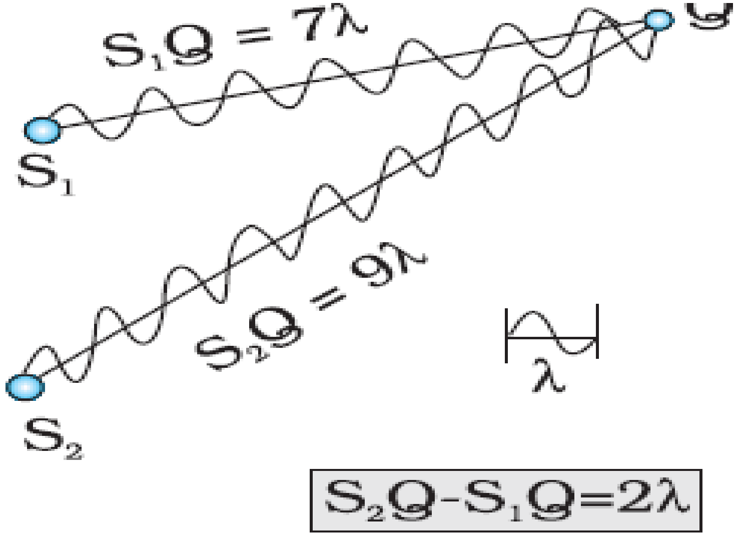
Study the line along
which constructive interference
takes place



$y_1 = a \cos \omega t$

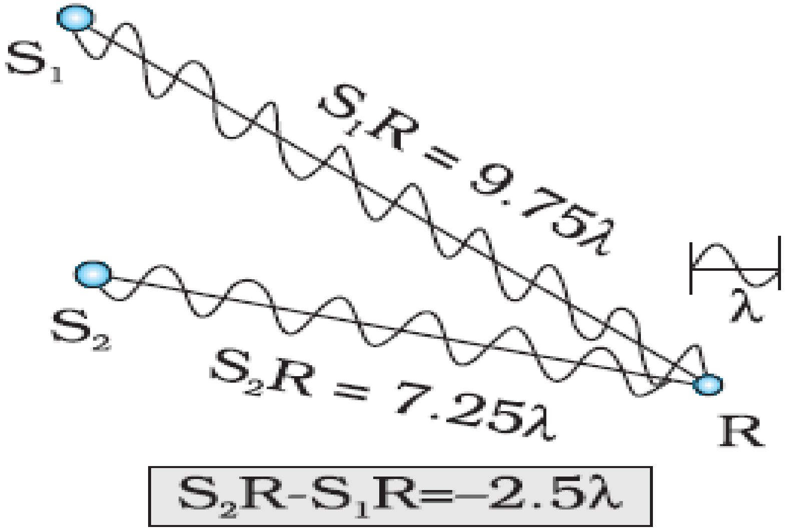
$y_2 = a \cos \omega t$

$y = y_1 + y_2 = 2\, a \cos \omega t$



(a)

Do the derivation



(b)

If we have two coherent sources S1 and S2 vibrating in phase, then for an arbitrary point P whenever the path difference,

$S1P \sim S2P = n\lambda$ $(n = 0, 1, 2, 3,...)$

we will have constructive interference and the resultant intensity will be $4I0$;

the sign ~ between S1P and S2 P represents the difference between S1P and S2 P. On the other hand, if the point P is such that

the path difference,

$S1P \sim S2P = (n+1/2)\lambda$ $(n = 0, 1, 2, 3, ...)$

we will have *destructive interference* and the resultant intensity will be zero.

$$S_2P - S_1P = n\lambda; \quad n = 0, 1, 2 \ldots$$

Now,

$$(S_2P)^2 - (S_1P)^2 = \left[D^2 + \left(x + \frac{d}{2}\right)^2\right] - \left[D^2 + \left(x - \frac{d}{2}\right)^2\right] = 2xd$$

where $S_1S_2 = d$ and $OP = x$. Thus

$$S_2P - S_1P = \frac{2xd}{S_2P + S_1P}$$

If x, $d \ll D$ then negligible error will be introduced if $S_2P + S_1P$ (in the denominator) is replaced by $2D$.

$d = 0.1$ cm, $D = 100$ cm, OP = 1 cm

(which correspond to typical values for an interference experiment using light waves),

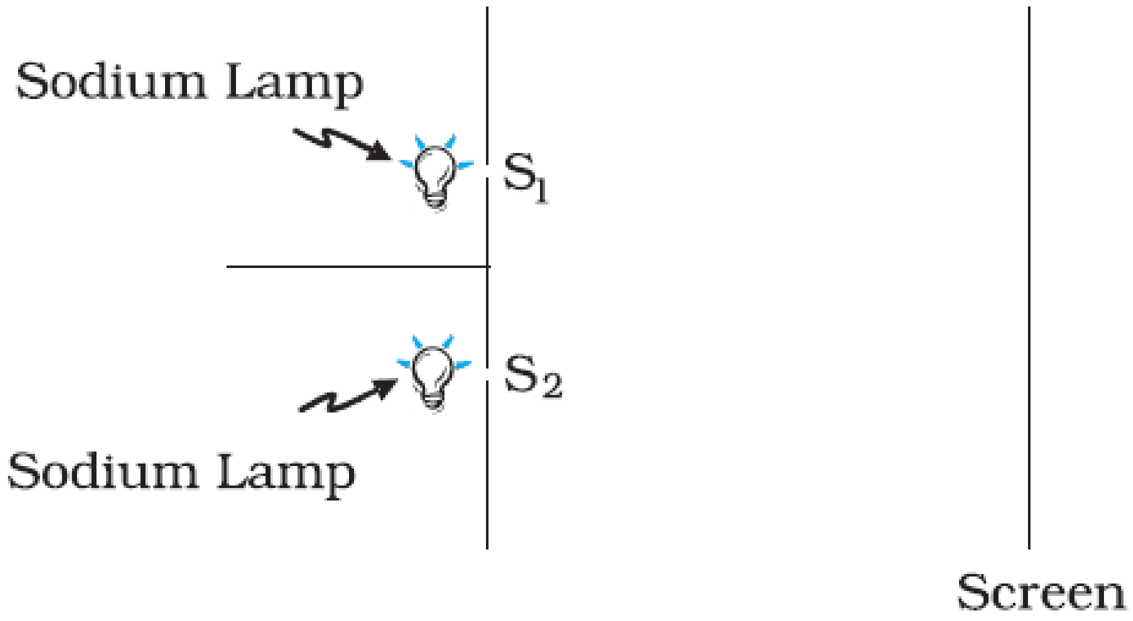
constructive interference resulting in a bright region when

$$x = x_n = \frac{n\lambda D}{d}\,;\; n = 0,\ \pm 1,\ \pm 2,\ \ldots$$

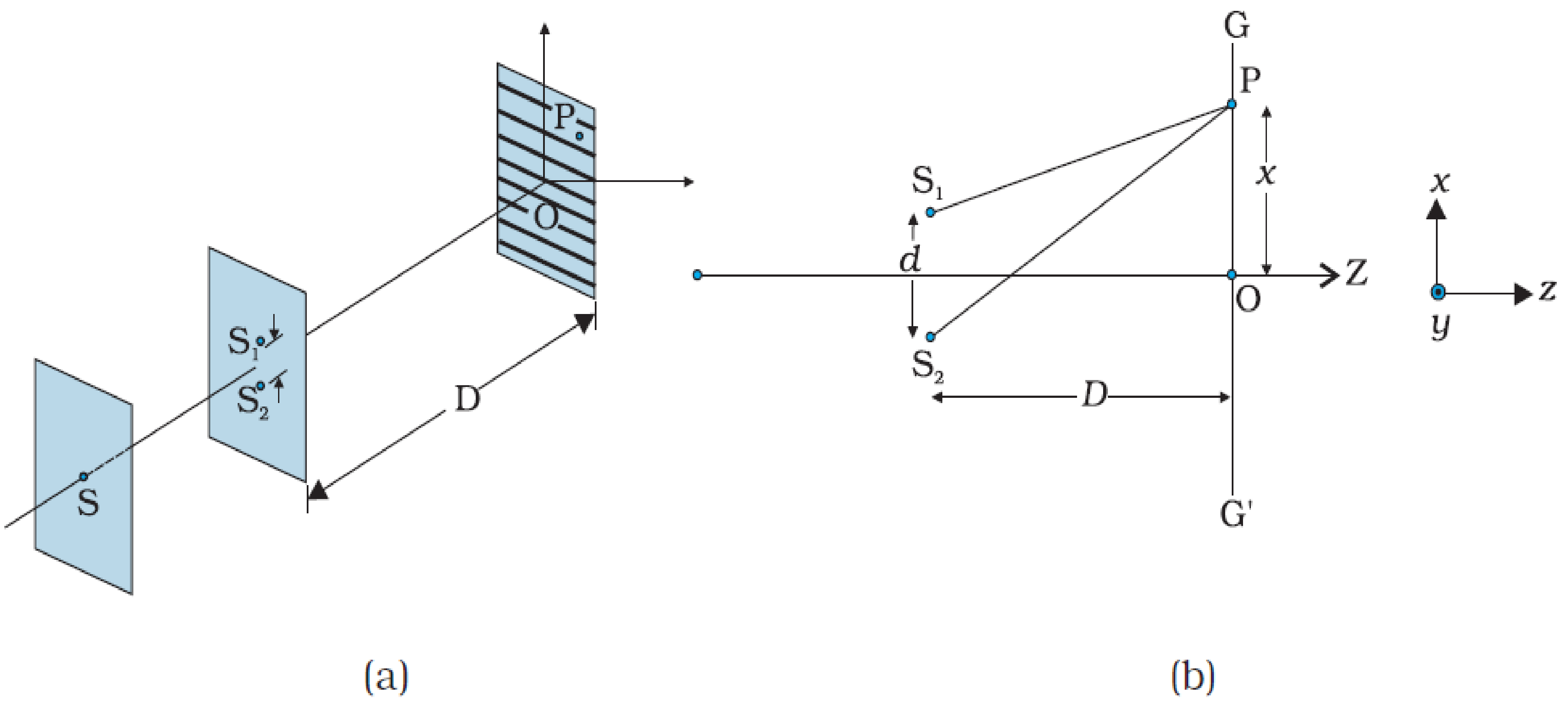
destructive interference resulting in a dark region when

$$x_n = \left(n + \frac{1}{2}\right)\frac{\lambda D}{d}; n = 0 \pm 1. \pm 2..$$

Why do we need coherent sources?
Why will there be no interference pattern on the screen ?

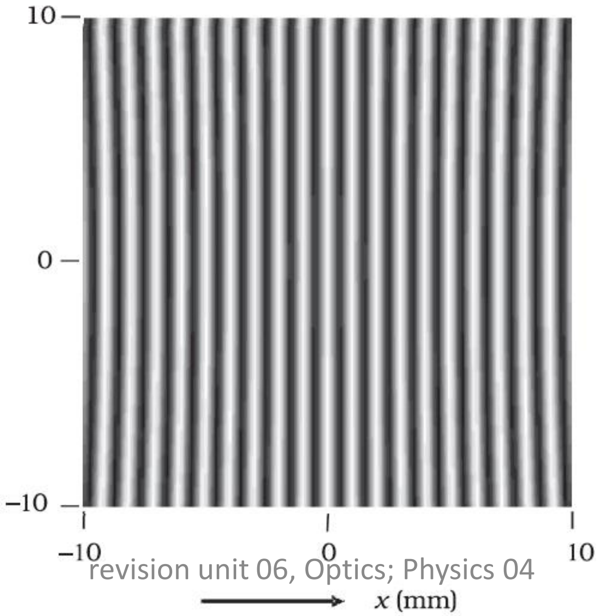


Young's double slit experiment



fringes

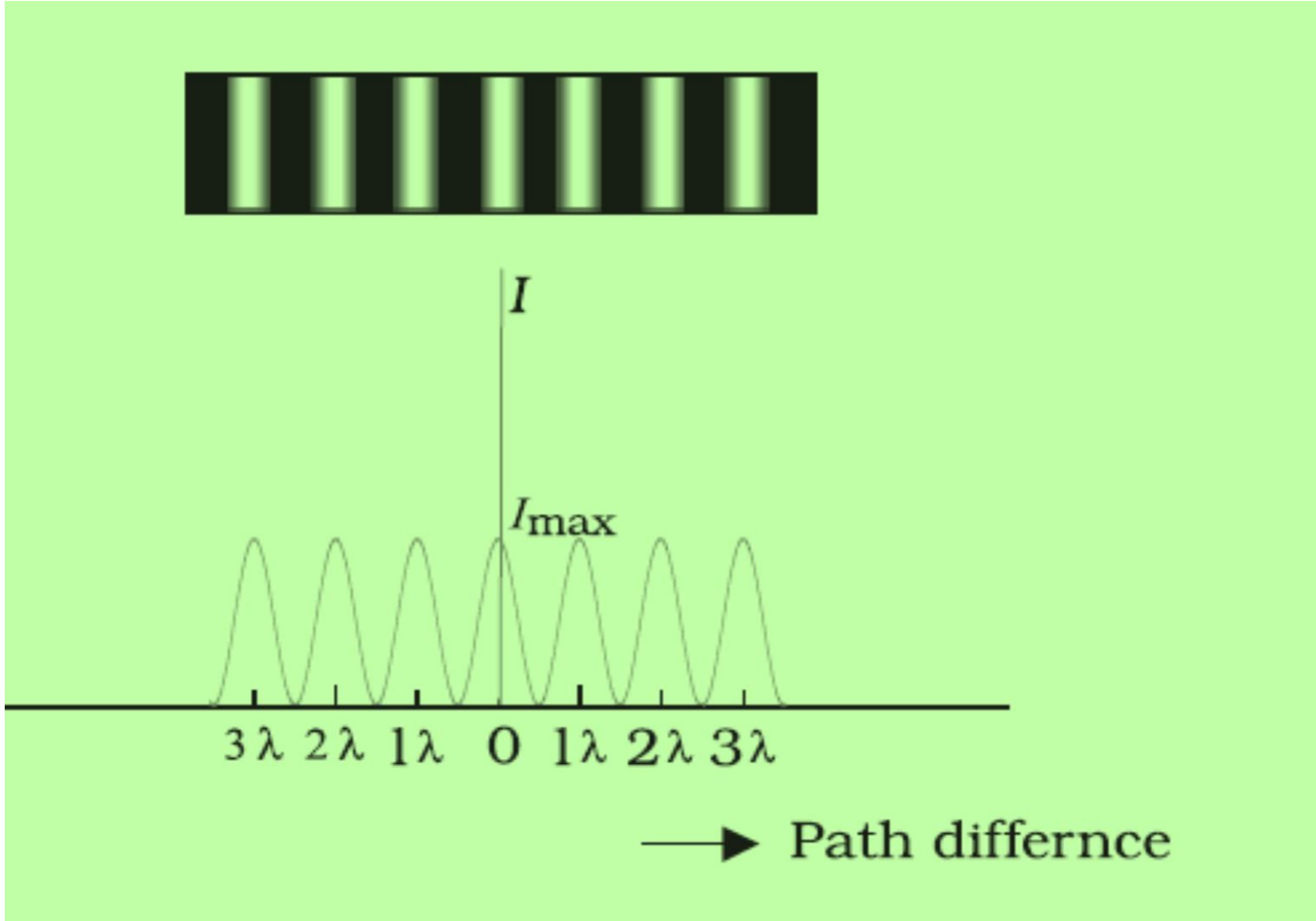
- Dark and bright bands appear on the screen
- Such bands are called *fringes*.
- Dark and bright fringes are equally spaced
- The distance between two consecutive bright and dark fringes is called fringe width



Fringe width depends upon

$$\beta = x_{n+1} - x_n$$

$$\text{or} \quad \beta = \frac{\lambda D}{d}$$



Examples:

1. **Two slits are made one millimeter apart and the screen is placed one meter away. What is the fringe separation when blue green light of wavelength 500 nm is used?**

Solution: fringe spacing = $\frac{D\lambda}{d} = \frac{1\times5\times10^{-7}}{1\times10^{-3}}$ m = **0.5 mm**

2. What is the effect on interference fringes in a Young's double-slit experiment due to each of the following operations:

- **The screen is moved away from the plane of the slits;**

- Angular separation of the fringes remain constant (= λ/d). The actual separation of the fringes increases in proportion to the distance of the screen from the pane of the two slits.

❑The separation between the two slits is increased;

- The separation of the fringes (and also angular separation) decreases.

❑The source slit is moved closer to the double-slit plane;

- Let s be the size of the source and S its distance from the plane of the two slits. For interference fringes to be seen, the condition $s/S < \lambda/d$ should be satisfied; otherwise, interference patterns produced by different parts of the source overlap and no fringes are seen. Thus, as S decreases (i.e. the source of the slit is brought closer), the interference pattern gets less and less sharp, and when the source is brought too close for this condition to be valid, the fringes disappear. Till this happens, the fringes separation remain fixed.

❑The width of the source slit is increased;

- As the source slit width increases, fringe pattern gets less and less sharp. When the source slit is so wide that the condition $s/S < \lambda/d$ is not satisfied, the interference pattern disappears.

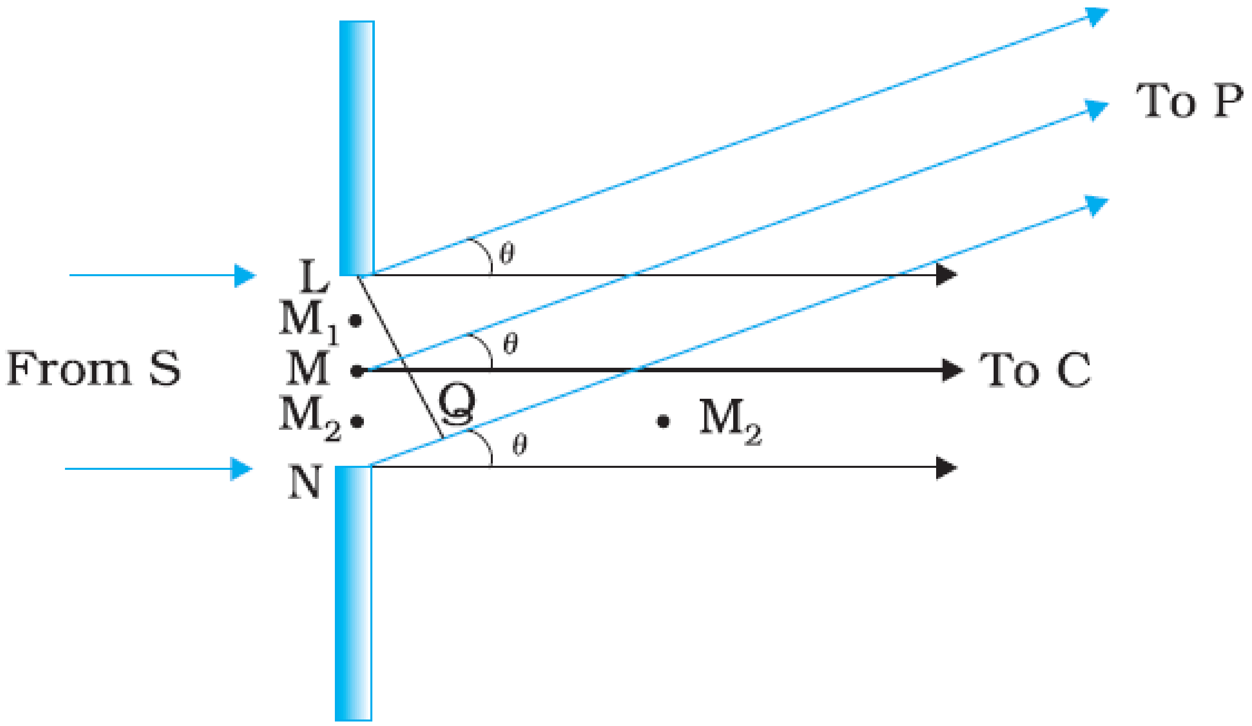
❑The monochromatic source is replaced by a source of white light?

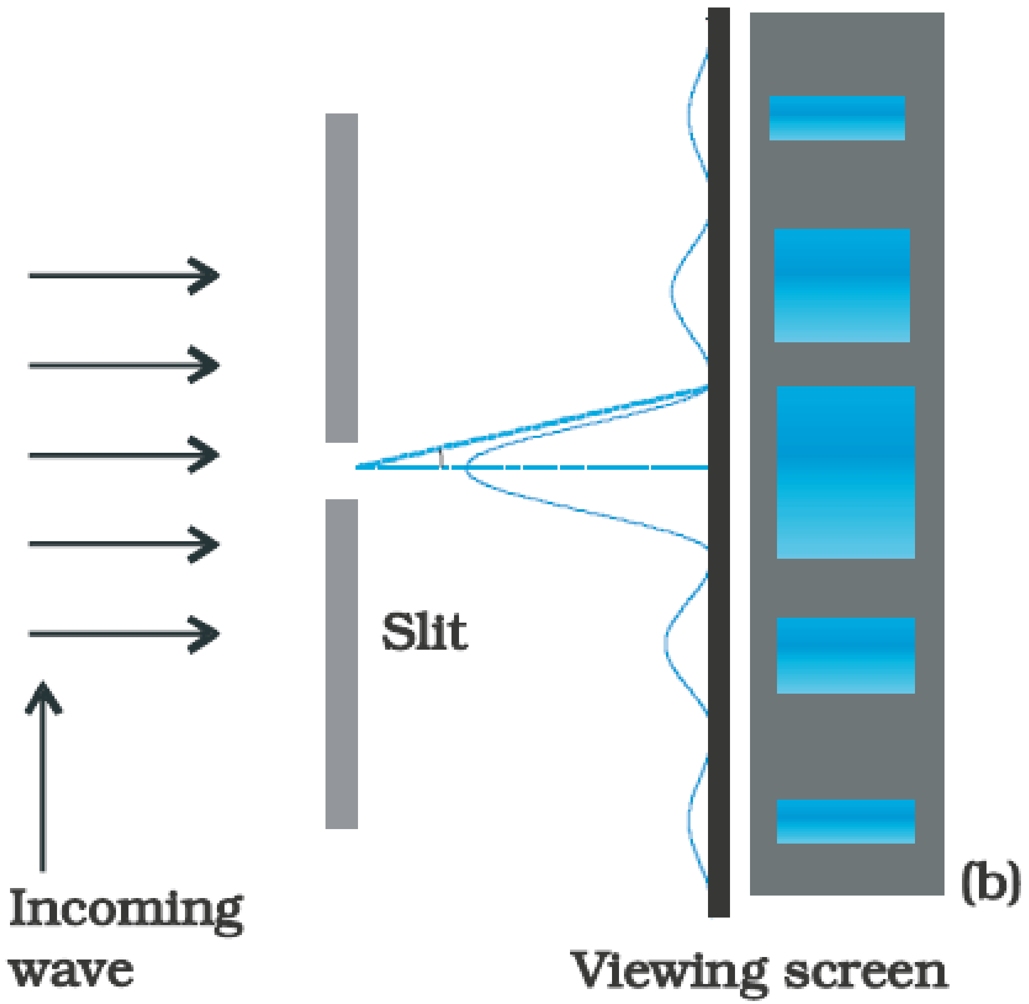
- The interference patterns due to different component colours of white light overlap (incoherently). The central bright fringes for different colours are at the same position. Therefore, the central fringe is white. The fringe closest on either side of the central white fringe is red and the farthest will appear blue. After a few fringes, no clear fringe pattern is seen.

❑The (monochromatic) source is replaced by another (monochromatic) source of shorter wavelength;

- The separation of the fringes (and also angular separation) decreases.

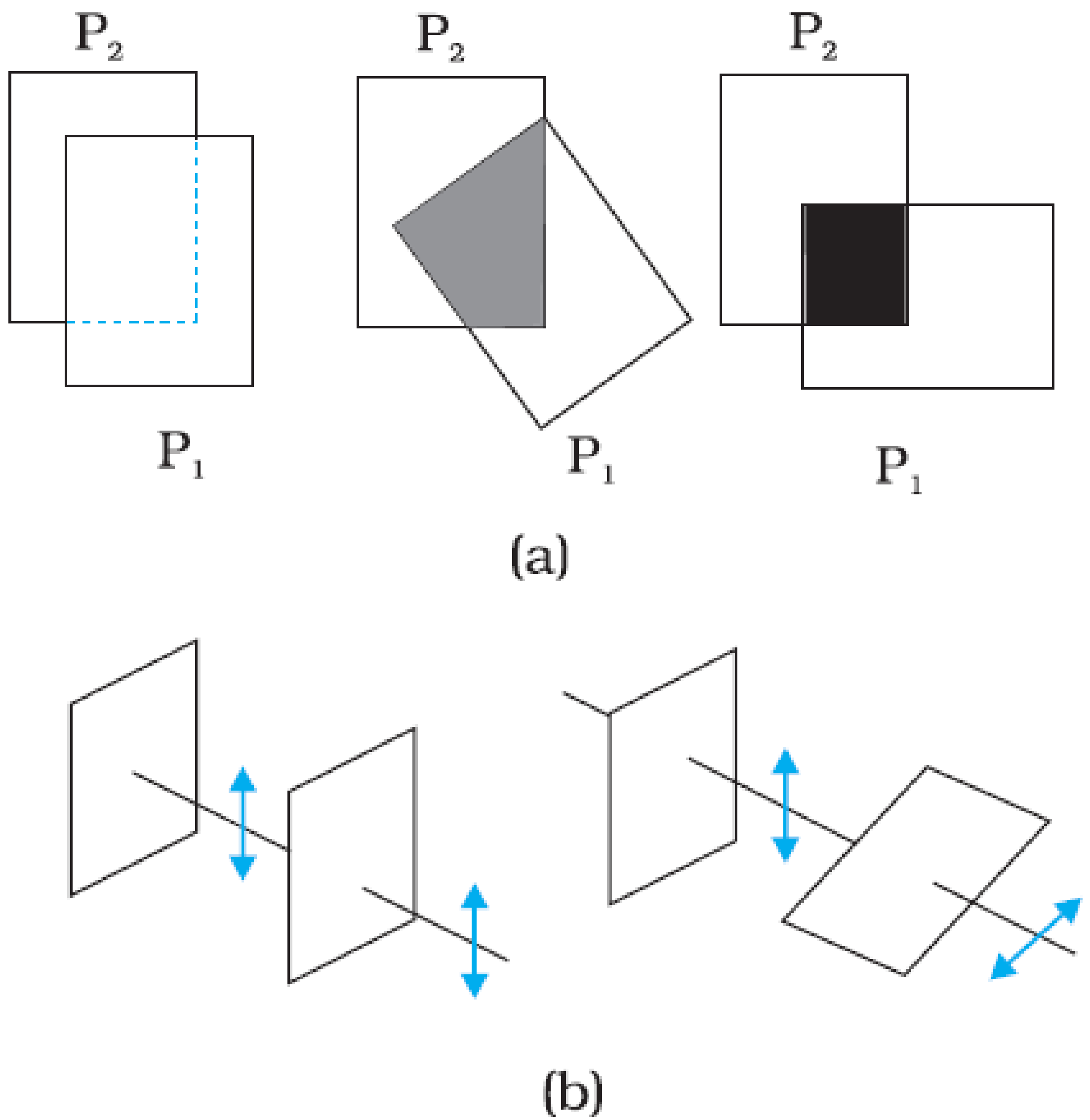
Diffraction




Slit
Incoming
wave
Viewing screen
(b)

Polarisation

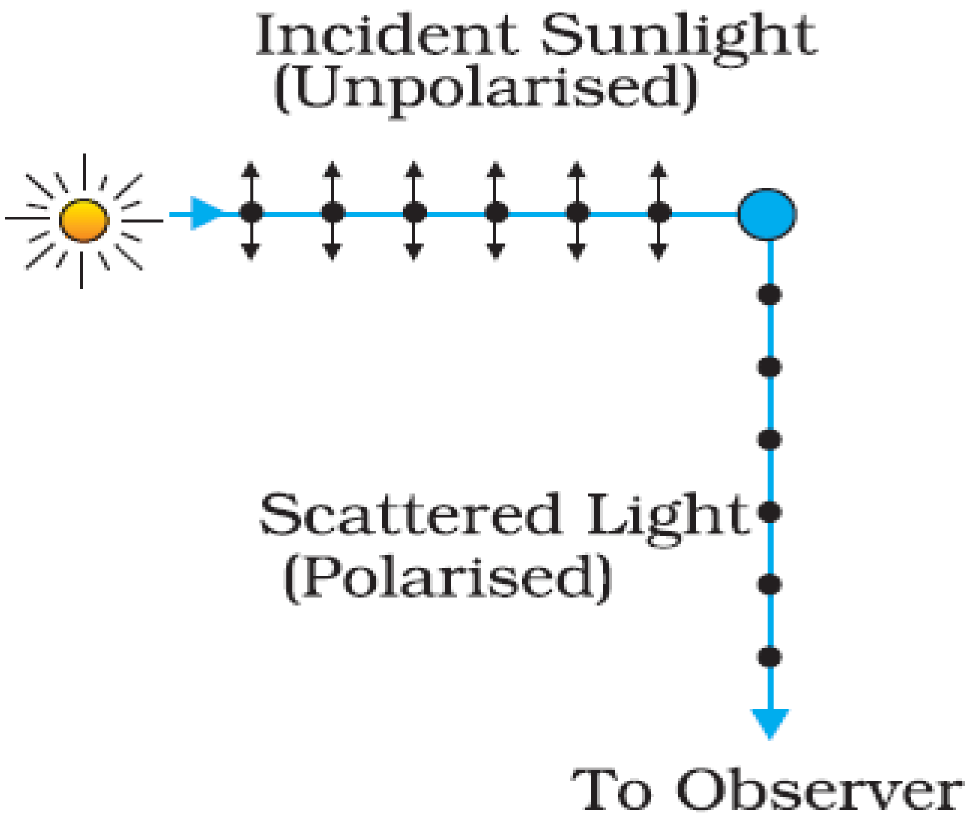
- Light waves are transverse in nature; i.e., the electric field associated
  with a propagating light wave is always at right angles to the direction of propagation of the wave.
- A polaroid consists of long chain molecules aligned in a particular direction.
- The electric vectors (associated with the propagating light wave along the direction of the aligned molecules get absorbed.
- So the electric vector is restricted to move in a particular plane
- Such light is called plane polarised light
- To check polarisation we need an analyser


P₂
P₁
(a)
(b)

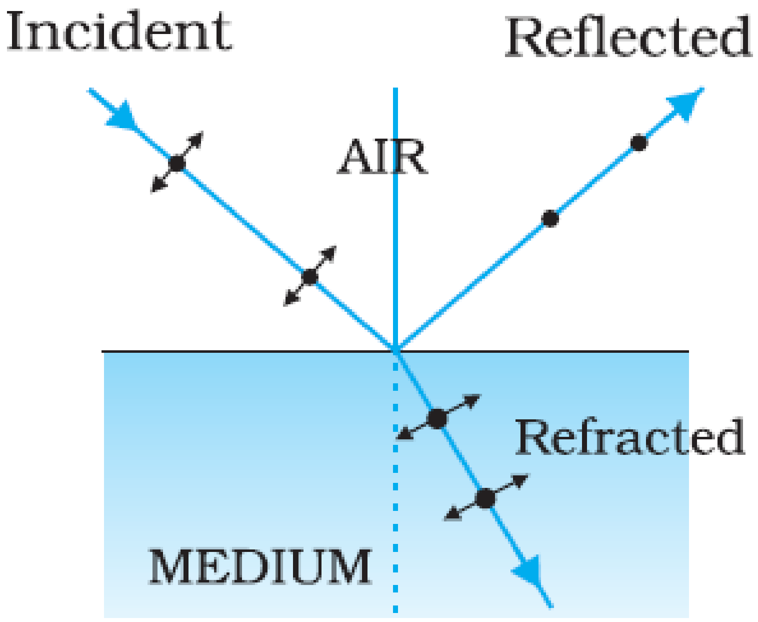
Malus' law

$$I = I_0 \cos^2\theta$$

Polarisation by scattering



Polarisation by reflection



Brewster's angle

- When reflected wave is perpendicular to the refracted wave,
- the reflected wave is a totally polarised wave.
- The angle of incidence in this case is called *Brewster's angle* and is denoted by i_B.

$$\mu = \frac{\sin i_B}{\sin r} = \frac{\sin i_B}{\sin(\pi/2 - i_B)}$$

$$= \frac{\sin i_B}{\cos i_B} = \tan i_B$$

Uses of Polaroids

- **Control the intensity in sunglasses windowpanes etc.**
- **Polaroids are also used in photographic cameras**
- **3D movie cameras and scanners**